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The Gravity and The Quantum: A Bohr-inspired Synthesis

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An effective angular momentum quantization condition of the form $mvr = n\hbar(m/m_F)$ is used to obtain a Bohr-like model of Hydrogen-type atoms and a modified Schrödinger equation. Newton's constant, G, of Gravitation gets explicitly involved through the fundamental mass m_F as defined in the sequel. This non-relativistic formalism may be looked upon as a "testing ground" for the more general synthesis of the gravity and the quantum.

One of the pivotal principles of physical nature underlying mathematical methods of the quantum theory can be said to be the following.

Quantization of a physical quantity is, at a very basic level, expressing that physical quantity as an integral multiple of "some unit" of that quantity, that unit being obtainable from the fundamental constants of Nature alone. This principle based on the dimensional analysis has a crucial role in not only the orthodox quantum theory but also in subsequent developments in Physics.

As is well known, Niels Bohr, in a masterly use of this principle, had postulated the quantization rule for the angular momentum of an electron (of mass m_e and charge e) orbiting an atomic nucleus (of atomic number Z) in his famous model of the hydrogen atom as [1]:

$$m_e v_n r_n = n\hbar$$

to express the angular momentum as an integer, n, multiple of Planck's constant, $\hbar$, a fundamental constant. As is also well known, having imposed this condition, the wave number of a photon that is emitted by the atom transiting from a state n_2 to a state n_1 ($n_2 > n_1$) is given by

$$\frac{1}{\lambda} = Z^2 R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where $R = e^4 m_e/2\hbar^2 c$ (c being the speed of light in vacuum) is the Rydberg constant, which now gets determined completely in terms of the fundamental constants of Nature.

The force of gravity between an atomic nucleus and an electron is about 10^{-42} times weaker than their electrostatic force of attraction. Still, it does seem somewhat unsatisfactory that Bohr's model does not incorporate basic constants other than Planck's constant, like Newton's constant G of gravitation, a fundamental constant.

If gravity and the quantum nature of the physical phenomena are to result from only a single theory, then it seems plausible that we will have some limited formulation of the quantum theory and, hence, of Bohr-like model of the atom in which G would also appear along with $\hbar$ and other fundamental constants of Nature. However, no such formulation is currently available.

Perhaps, this lacuna can be remedied by using a "unit" for the angular momentum different than $\hbar$ that was used by Bohr. Needless to say, but we still wish to emphasize at this place that, any new unit of the angular momentum must be constructible out of the basic constants.

In this paper, we show that this can indeed be done. We also provide a Schrödinger-like equation for the formulation of corresponding ideas. Just as Bohr's model provided the testing ground for developments in the orthodox quantum theory, the proposed Bohr-like model may be a testing ground for the general synthesis (*eg*, [2, 3, 4, 5, 6]) of the gravity and the quantum.

To begin with, we note that using c, h, G, m_e and e, we can form a quantity that has the dimensions of the angular momentum as:

$$A_P = e^2\, m_e \sqrt{\frac{G}{c^3 h}} \approx 4.05 \times 10^{-53} \text{ cgs units.} \quad (1)$$

It is however very tiny. Notably, Planck's constant $\hbar$ is 2.61×10^{25} times larger than it.

Therefore, to be able to express the "usual" angular momentum of an atomic electron in terms of A_P, we "scale" this tiny unit by a factor Υ and express Bohr's condition of the quantization of the angular momentum as

$$m_e v_n r_n = n \Upsilon A_P = n\, \Upsilon e^2\, m_e \sqrt{\frac{G}{c^3 h}} \quad (2)$$

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We could strongly object to the occurrences of e and m_e in (1) since these are not fundamental constants of Nature as are G, c and h, but are particle-specific quantities. However, we also note that (2) is equivalent to expressing the linear momentum of an atomic electron as:

$$m_e v_n = \frac{Z}{n\Upsilon}\sqrt{\frac{c^3 h}{G}} \tag{3}$$

where the square-root is related to Planck's length $L_p = \sqrt{hG/c^3}$ by h/L_p.

Evidently, e and m_e do not at all play any role in condition (3). Essentially, the quantization condition for one physical quantity implies the quantization of another related physical quantity with the proportionality factors of one affecting those of the other [12].

In (3), we have chosen proportionality constants to "match" the formulas of the present study with those of the "usual" atomic physics. Thus, the factor of $Z/n\Upsilon$ is "adjustable" in the condition (3), we may then note here.

It then follows that the radius of the electronic orbit of the atom is given by

$$r_n = \frac{n^2\Upsilon^2}{Z}\frac{e^2 m_e G}{c^3 h} \tag{4}$$

and that the wave number of a photon, which is emitted by the atom transiting from a state n_2 to a state n_1 ($n_2 > n_1$), is given by:

$$\frac{1}{\lambda} = \frac{Z^2 c^2}{2\Upsilon^2 G m_e}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \tag{5}$$

It now immediately follows that, for an appropriate value of Υ, the same atomic physics would result even with the newly chosen unit of the angular momentum.

On its face value, the above exercise appears to be only of pure academic interest and, hence, not useful for further progress, if the same atomic physics as that of Bohr's model is to result from it. But, we would like to note that the validity of the proposed model can be beyond that of Bohr's model using Planck's constant simply because Υ is a free parameter.

We also note that the validity condition for the non-relativistic treatment reads:

$$v_n/c \ll 1 \Rightarrow \ \Upsilon^{-1} \ll \frac{nm}{Z}\sqrt{\frac{G}{ch}} \approx \frac{nm}{Z} \times 5.6 \times 10^3$$

which, for an electron revolving around the usual atomic nucleus, is

$$v_n/c \ll 1 \Rightarrow \ \Upsilon^{-1} \ll \frac{n}{Z} \times 5.1 \times 10^{-23}$$

For comparison with Bohr's model, we note that the condition for non-relativistic treatment reads $(Z/n)\,\alpha << 1$ and that this is independent of the mass of the particle under consideration [13]. The first of our inequalities, with mass, can be satisfied for suitable values of m, e and Υ.

After having demonstrated the plausibility of the Bohr-like model [14] of the atom involving the new unit of angular momentum or, equivalently, the new unit of momentum, we turn to aspects related to ideas of Schrödinger.

Now, as is well known [1], Schrödinger's equation is obtained from $E = p^2/2m + V$, where p is the momentum and V is the potential, by substituting $p \to -\imath\hbar\nabla$ and $E \to \imath\hbar\partial/\partial t$ wherein Planck's constant $\hbar$ serves to keep the dimensions of the terms correct. (All the "undeclared" symbols will have their usual meaning.)

In the above spirit, we now use the "new unit" (that has the dimensions of Planck's constant) and make the obvious substitutions $p \to -\imath A_P \nabla$ and $E \to \imath A_P \partial/\partial t$ in $E = p^2/2m_e + V$. This leads us to the following Schrödinger-like equation:

$$- \ \frac{\Upsilon e^2 m_e}{\imath}\sqrt{\frac{G}{c^3 h}}\frac{\partial\Psi}{\partial t} = - \ \frac{e^4 m_e \Upsilon^2}{2}\,\frac{G}{c^3 h}\,\nabla^2\Psi + \ V\Psi$$

For a free particle, this equation reduces to

$$\imath\frac{\partial\Psi}{\partial t} = -\,\frac{e^2\Upsilon}{2}\sqrt{\frac{G}{c^3 h}}\,\nabla^2\Psi$$

Notice that m_e drops out of the equation. The factor Υ then essentially determines the "fundamental" mass, m_F, as

$$m_F = \frac{1}{2\pi\Upsilon e^2}\sqrt{\frac{c^3 h^3}{G}} \ = \ \frac{1}{\alpha\Upsilon}\sqrt{\frac{ch}{G}} \ = \ \frac{0.02374}{\Upsilon}\ \mathrm{gm} \tag{6}$$

where $\alpha = e^2/c\hbar \approx 1/137$ is the well known fine structure constant.

With this definition, the Schrödinger equation given above can be written as:

$$-\imath\hbar\left(\frac{m}{m_F}\right)\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{m}{m_F}\right)^2\nabla^2\Psi + V\Psi \tag{7}$$

wherein we dropped the subscript e on the mass m, as this equation holds generally.

With the definition (6), the Bohr quantization condition (2) then reads $m_e v_n r_n = n\hbar(m_e/m_F)$, while the quantization condition (3) reads $m_e v_n = (Z/n)\alpha m_F c = (Z/n)(e^2 m_F/\hbar)$. It should then be noticed that we only have "scaled", by m/m_F,

Planck's constant of the usual Schrödinger's equation, and also that no new solutions are implied here. Clearly, standard solutions of Schrödinger's equation can be used to explore the consequences of the existence of m_F in (7).

We have then obtained here, only a preliminary, Bohr-inspired, synthesis of the gravity and the quantum by way of the fundamental mass m_F and the Schrödinger equation (7) at the non-relativistic level of its applications.

As closing remarks, one of us (SMW) would like to comment as follows.

Evidently, we will recover, as was stated before, the "usual" atomic physics when $m = m_F = m_e$. (In this case, $\Upsilon = (\alpha m_e)^{-1}\sqrt{ch/G}$.) The proposed preliminary synthesis of gravity with the quantum then incorporates the usual explanations of quantum phenomena.

An apparent limitation of (6) and, hence, of (7) is that these involve charge, *ie*, in (6), $e \neq 0$. But, this is due to the following situation.

Notice here that c^3/GX^2 has the dimensions of Planck's constant if X has the dimensions of $1/L$, and $1/L$ are the dimensions of Rydberg's constant. Thus, no quantity of the dimensions of angular momentum, other than $\hbar$, can be constructed from G and c without involving $\hbar$ or any particle-specific quantities like charge and mass.

Now, momentum has the dimensions of $[\hbar]/L$. Hence, similar to the above case with the angular momentum, no quantity having dimensions of momentum can be constructed out of G and c without explicitly involving $\hbar$ or any particle-specific quantities like charge and mass.

But, $e^2 m_F$ is completely determined by fundamental constants G, c, $\hbar$, and the electronic charge e appears in the form of the fine structure constant only. Thus, the appearance of e in (6) is not any serious limitation.

Then, the appearance of e in (6) rather points at a way in which equations of a more general formulation could be expected to reduce to their non-relativistic forms. Of course, unless a satisfactory mathematical framework is at hand for the general synthesis of gravity and the quantum, it is difficult to elaborate on as to how this reduction of general equations to (7) obtains.

It is however clear that we have only two possibilities in the non-relativistic domain: either the usual Schrödinger equation or the equation (7). As stated before, equation (7) is "equivalent" to the usual Schrödinger's equation, albeit with "modified" Planck's constant. The modified Planck's constant will arise unless of course $m = m_F$, *ie*, if $\Upsilon = (\alpha m)^{-1}\sqrt{ch/G}$ always. In this last case, the onus then falls on the more general formulation of the synthesis of the gravity with the quantum to explain as to why an entirely arbitrary constant Υ always has this form!

As this last possibility appears preposterous, we look upon an equation of the form (7) as the "proper" quantum equation of the non-relativistic domain in that equations of a more general formulation should reduce to (7). The following is supportive of this view.

We then notice that if the fundamental mass m_F were taken to be dependent on different powers of the fine structure constant α by demanding that $\Upsilon \propto 1/\alpha^N$ (N - an integer), *ie*, if $m_F = \alpha^{N-1}\sqrt{ch/G}$, then we obtain a spectrum of fundamental mass values for the chosen value of charge e. (Different such powers of α may be expected to arise in a synthesis of gravity with the quantum more general than the present one.)

Another possibility is of an integral multiple of the fundamental mass m_F as a possible origin of the mass spectrum. In this case, Υ could be chosen to yield the right mass for one particle, say, π^o-meson. Then, the observed [7] mass spectrum of some of the elementary particles is "explainable" within the present formalism.

The so-obtained mass spectra should then turn out be relevant for the mass spectrum of elementary particles [7]. We would like to note however that the equation (7) does not provide any "explanation" for the masses of elementary particles or even the mass spectrum.

In this connection, we would like to also note that important effects of spin and other characteristics of elementary particles are not included in the present simple analysis. The aforementioned mass spectrum should therefore be looked upon only as a feature of the proposed synthesis. As compared to the usual quantum formulation, the equation (7) then has quite a "direct" relevance to the non-relativistic behavior of each member of the aforementioned mass spectrum.

Now, this synthesis of the gravity and the quantum can be expected to have observable consequences. In particular, interesting situations [15] could be expected to arise when $m \neq m_F$ in (7). We then look for "quantum" situations, *eg*, those considered in [8], wherein the "effective" value of Planck's constant could be "different" than that in, *eg*, Planck's radiation formula.

Of some particular interest in this connection is the phenomenon of Bloch oscillations [9, 10] of electrons in a crystal. Although it has been difficult to observe Bloch oscillations in natural crystals, recent advances in laser spectroscopy have made it possible to observe [11] the Bloch oscillations of ultra-cold atoms in optical lattices generated by interfering laser beams trapping atoms.

For laser generated optical lattices, the Bloch frequency is given by $\nu_B = mg\lambda/2h$ where λ is the frequency of laser light producing the interference that traps atoms of mass m in a uniform gravitational field of acceleration g. (This is based on standard Schrödinger's equation.) It should then be possible to "create" situations wherein the "effective" Planck's constant should be experimentally observable confirming or refuting thereby the proposed synthesis of gravity and the quantum at the non-relativistic level.

We would like to reemphasize that any non-observation of the effective Planck's constant, as implied by the present study, will be a certain theoretical surprise: we will have to understand then as to why Nature has selected an arbitrary constant Υ only in a particular way.

Lastly, we would like to note the following. Any conceptual and mathematical framework of the unification of all the fundamental interactions will have to necessarily incorporate all the fundamental physical constants along with particle-specific physical variables such as mass and charge of the particle. In suitable approximation, whose manner is unclear at the present, we will have to recover the equation (7) from the mathematical formalism of the theory of unification. This is of course subject to the aforementioned caveat that we infer the existence of the effective Planck's constant in some experimental situation.

Clearly, this preliminary synthesis of the gravity with the quantum already has consequences for general physical situations and for a fundamental theory of the unification of all the physical interactions, both. Although it is quite an elementary and, hence, limited formalism, its potential implications appear to be fundamental to further progress. It would therefore be important to experimentally establish the existence of an effective Planck's constant as in (7).

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[12] For Bohr's model using Planck's constant as a unit of the angular momentum, the linear momentum is given as $m_e v_n = Ze^2 m_e/n\hbar = (Z/n)\alpha m_e c$. Proportionality factors have been adjusted to get Z/n for the momentum in (3). It is then that the formula for the wave number of emitted radiation has the form of (5).

[13] This is what leads us to doubt the validity of the non-relativistic treatment for "strange atoms" like μ-meson atoms , as have been considered in many textbooks. Our work here implies however that the non-relativistic treatment can be applicable in certain of such cases.

[14] AHW worked out this Bohr-like model following the suggestion of SMW of exploring the changes to Bohr's model with the condition (3) in the form $m_e v_n = \beta\sqrt{c^3 h/G}$ with β as a dimensionless proportionality constant.

[15] We propose to explore different such observable consequences of m_F in a later work.